ENGINEERING CALCULATION OF THERMOELASTIC STRESSES IN MULTICHANNEL-TYPE COMPOSITE STRUCTURES

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The article presents the example of an engineering calculation of the thermal stresses in multichannel-type composite structures strengthened by heat treatment.

The technological process of producing composite structures by explosive welding is accompanied by the formation of residual stresses in the material. Their intensity can be reduced while the mechanical properties of the structure are improved at the same time by heat treatment; however, this in turn entails new thermal stresses. Their calculation in real bodies on the basis of exact methods of mathematical thermoelasticity is a very complex problem. The present article suggests an engineering method of calculating thermal stresses in multichannel-type structures illustrated in Fig. 1. It is based on the following assumptions: the temperature and the state of stress and strain do not change along the structure; the stiffening ribs are subjected solely to tension (compression) by the load q_3 which we will later determine; the temperature in the layers changes linearly (for the case of symmetric section of the profile of the base and symmetric boundary conditions).

This last assumption may interfere with the condition of continuity. However, Manso [1] showed this has an imperceptible effect on the stress distribution, and engineering calculations may be based on it.

The symmetry of the temperature field relative to the profile of the base of the structure justifies that as object for calculation, not the entire section of the multilayered strip is selected but only its upper or lower part (Fig. 1).

The problem examined here was solved on the assumption that the temperatures were specified on the upper or lower side of the calculation element, and also in the zone of the joint [2]. In contrast to [2] we assume that at the place of the weld the heat fluxes are specified, and therefore the temperature in the zone of the joint is determined with the aid of the equality

$$\lambda_T^1 \frac{\partial \tau_1}{\partial y}\bigg|_{y=+0} = \lambda_T^2 \frac{\partial \tau_2}{\partial y}\bigg|_{y=-0},$$

$$\tau_1|_{y=+0} = \tau_2|_{y=-0},$$

λ_T^i are the thermal conductivities.

We write Hooke's law for the case of plane strain as follows:

$$\sigma_{xx}^i \equiv \sigma_{xx}^{i*} - \alpha_T^i \tau_i, \quad \sigma_{yy}^i \equiv \sigma_{yy}^{i*} - \alpha_T^2 \tau_i, \quad \sigma_{yx}^i \equiv \sigma_{yx}^{i*}, \tag{1}$$

$$\tau_1 = \frac{T_1 - T_0}{h_1} y + T_0, \quad \tau_2 = \frac{T_0 - T_2}{h_2} y + T_0,$$

where $\sigma_{xx}^{i*} = \lambda_i \Theta + 2\mu_i e_{xx}$; $\sigma_{yy}^{i*} = \lambda_i \Theta + 2\mu_i e_{yy}$; $\sigma_{yx}^{i*} = 2\mu_i exy$; $\Theta = e_{xx} + e_{yy}$; $\mu_i = E_i/2(1+\nu_i)$; $\lambda_i = E_i\nu_i/(1+\nu_i)\cdot(1-2\nu_i)$; τ_i is the temperature difference between the metal and the cooling medium. Then the equations of equilibrium assume the form

$$\frac{\partial \sigma_{xx}^{i*}}{\partial x} + \frac{\partial \sigma_{xy}^{i*}}{\partial y} - \frac{\partial(\alpha_T^i \tau_i)}{\partial x} = 0, \tag{2}$$

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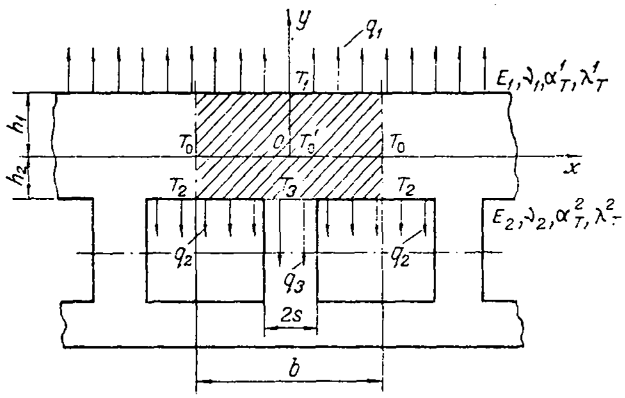

Fig. 1. Calculation element of composite structures.

$$\frac{\partial \sigma_{xy}^{i*}}{\partial x} + \frac{\partial \sigma_{yy}^{i*}}{\partial y} - \frac{\partial (\alpha_T^i \tau_i)}{\partial y} = 0.$$

If on the upper and lower boundaries of the calculation element the temperature is specified, then the following boundary conditions and conditions of conjugation are added to Eqs. (2):

$$y = + h_1, \quad \sigma_{yy}^1 = 0 \qquad \sigma_{yy}^{1*} = \alpha_T^1 T_1$$

$$-\frac{b}{2} < x < \frac{b}{2}, \ \sigma_{xy}^1 = 0 \quad \Rightarrow \quad \sigma_{xy}^{1*} = 0 \quad ;$$

$$y = - h_2 \qquad \sigma_{yy}^{2*} = \alpha_T^2 T_2,$$

$$-\frac{b}{2} < x < - s, \ s < x < b/2, \quad \sigma_{xy}^{2*} = 0;$$

$$y = 0 \quad u_1^* = u_2^*; \ v_1^* = v_2^*;$$

$$\sigma_{xy}^{1*} = \sigma_{xy}^{2*}, \ \sigma_{yy}^{1*} = \sigma_{yy}^{2*}.$$

The Lame equation is:

$$\mu_i \Delta^2 u^2 + \mu_i \left(\frac{1+\nu_i}{1-\nu_i}\right) \frac{\partial \Theta}{\partial x} + X_i = 0,$$

$$\mu_i \Delta^2 v^2 + \mu_i \left(\frac{1+\nu_i}{1-\nu_i}\right) \frac{\partial \Theta}{\partial y} + Y_i = 0.$$

Formulas (1), (2) make it possible to provide a force analogy for the temperature problem [1, 3] where the part of the volumetric forces will be played by the terms

$$X_i = -\frac{\partial(\alpha_T^i \tau_i)}{\partial x}, \quad Y_i = -\frac{\partial(\alpha_T^i \tau_i)}{\partial y}.$$

On the basis of this, the calculation of a composite structure as a whole reduces to solving the problem of determining the state of stress and strain of a laminated strip loaded on one side by the uniformly distributed load q_1, on the other side by the piecewise constant loads q_2 and q_3. The load q_3 takes into account the effect of the influence of the stiffening ribs on the state of stress and strain of the skin of the composite structure, and q_1 and q_2 the influence of the temperature:

$$q_i = R_i G_i \tau_i, \ i = 1, 2, \text{where } R_i = E_i / G_i (1 - \nu_i).$$

We put $q_3 = G_2 R_2 T_3$, where T_3 is the temperature at the boundary of the stiffening ribs. It can be determined from the condition of equilibrium of the calculation element under the effect of the surface and volumetric forces applied to it. It follows from the condition of equilibrium of an isolated element that

$$q_3 \cdot 2s + q_2 (b - 2s) - \frac{\partial \alpha_T^2 \tau_2}{\partial y} = q_1 b - \frac{\partial \alpha_T^1 \tau_1}{\partial y} b,$$

$$q_3 = q_2 + \frac{q_1 - q_2}{2s} b + \alpha_T^2 \frac{T_0 - T_2}{h_2} \frac{b - 2s}{2s} + \alpha_T^2 \frac{T_0^1 - T_3}{h_2} - \alpha_T^1 \frac{T_1 - T_0}{h_1} \frac{b}{2s}.$$

Then we obtain the following system of algebraic equations for determining T_0, T_3, T_0':

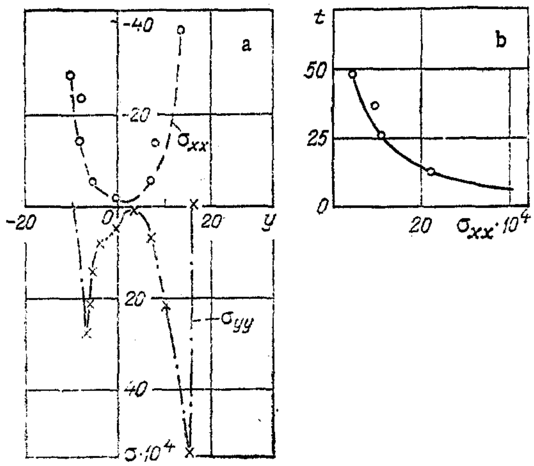

Fig. 2. Distribution of thermal stresses: a) σ_{xx} (N/m²) and σ_{yy} (N/m²) over the section of a composite structure 55 sec after the onset of cooling, x = 0; b) depending on the cooling time t (sec), x = 0.

$$T_0 = \frac{\lambda_T^1 T_1 h_2 + \lambda_T^2 T_2 h_1}{\lambda_T^2 h_1 + \lambda_T^1 h_2},$$

$$T_0^1 = \frac{\lambda_T^1 T_1 h_2 + \lambda_T^2 T_3 h_1}{\lambda_T^2 h_1 + \lambda_T^1 h_2},$$

$$\left(\frac{\alpha_T^2}{h_2} + E_2 G_2\right) T_3 = q_2 + \frac{q_1 - q_2}{2s} b + \alpha_T^2 \frac{T_0 - T_2}{h_2} \frac{b - 2s}{2s} + \alpha_T^2 \frac{T_0^1}{h_2} - \alpha_T^1 \frac{T_1 - T_0}{h_1} \frac{b}{2s}.$$

In accordance with [4] we seek the solution in the form

$$k = 2\pi n / b,$$

$$u_i^* = -\sum_{n=1}^{\infty} k [A_{ni} k \operatorname{sh} ky + B_{ni} k \operatorname{ch} ky + C_{ni} (\operatorname{ch} ky + ky \operatorname{sh} ky) +$$

$$+ D_{ni} (\operatorname{sh} ky + ky \operatorname{ch} ky)] \sin kx - \frac{\nu_i - 2}{4(\nu_i - 1)\mu_i} (K_i x + L_i xy),$$

$$v_i^* = \sum_{n=1}^{\infty} k [A_{ni} k \operatorname{ch} ky + B_{ni} k \operatorname{sh} ky + C_{ni} (ky \operatorname{ch} ky - 2 \operatorname{sh} ky) + D_{ni} (ky \operatorname{sh} ky - 2 \operatorname{ch} ky)] \cos kx$$

$$+ \frac{\nu_i}{4(\nu_i - 1)} \left(K_i y + \frac{L_i}{2} y^2\right) - \frac{3\nu_i - 2}{8(\nu_i - 1)\mu_i} L_i x^2 - \frac{R_i (T_0 - T_i)}{2\nu_i h_i} y;$$

$$\sigma_{xx}^{i*} = -E_i \sum_{n=1}^{\infty} k^2 [A_{ni} k \operatorname{sh} ky + B_{ni} k \operatorname{ch} ky + C_{ni} (\operatorname{ch} ky + ky \operatorname{sh} ky) + D_{ni} (\operatorname{sh} ky + ky \operatorname{ch} ky)] \cos kx$$

$$- \frac{E_i}{1 - \nu_i^2} \left\{ [K_i + L_i y] \left(\frac{\nu_i - 2}{4(\nu_i - 1)\mu_i} + \frac{\nu_i^2}{4(\nu_i - 1)} \right) - \frac{R_i (T_0 - T_i)}{2 h_i \nu_i} \right\},$$

$$\sigma_{xy}^{i*} = E_i \sum_{n=1}^{\infty} k^2 [A_{ni} k \operatorname{ch} ky + B_{ni} k \operatorname{sh} ky + C_{ni} ky \operatorname{ch} ky + D_{ni} ky \operatorname{sh} ky] \sin kx - L_i x,$$

$$\sigma_{yy}^{i*} = E_i \sum_{n=1}^{\infty} k^2 [A_{ni} k \operatorname{sh} ky + B_{ni} k \operatorname{ch} ky + C_{ni} (ky \operatorname{sh} ky - \operatorname{ch} ky) + D_{ni} (ky \operatorname{ch} ky - \operatorname{sh} ky)] \cos kx$$

$$+ \frac{E_i}{1 - \nu_i^2} \left\{ [K_i + L_i y] \left(\frac{\nu_i}{4(\nu_i - 1)} - \frac{\nu_i (\nu_i - 2)}{4(\nu_i - 1)\mu_i} \right) - \frac{R_i (T_0 - T_i)}{2 h_i \nu_i} \right\}.$$

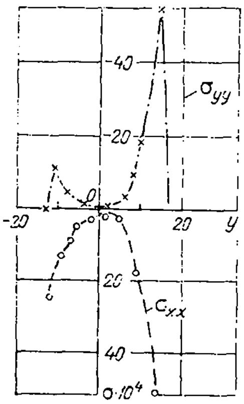

Fig. 3.

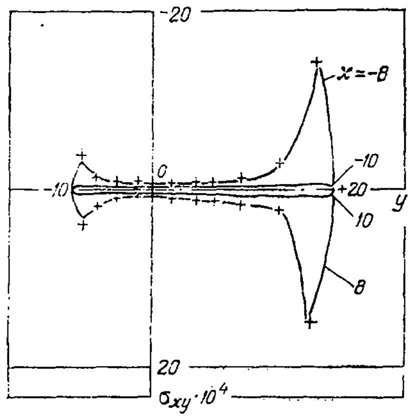

Fig. 4.

Fig. 3. Distribution of thermal stresses σ_{xx}, σ_{yy} over the section of a composite structure 50 sec after the onset of its cooling, x = ±8.

Fig. 4. Distribution of the thermal stresses σ_{xy} (N/m^2) over the section of a composite structure 50 sec after the onset of its cooling.

From the boundary conditions and the conditions of conjugation we obtain such a system for determining the unknown coefficients A_{ni}, B_{ni}, C_{ni}, D_{ni}, K_i, L_i (i = 1, 2):

$$\frac{E_2}{1-\nu_2^2}\left[\frac{\nu_2}{4(\nu_2-1)}(K_2-L_2h_2)+\frac{R_2(T_0-T_2)}{2\nu_2h_2}-\frac{\nu_2(\nu_2-2)}{4(\nu_2-1)\mu_2}(K_2-L_2h_2)\right]=\alpha_T^2T_2, \qquad (3)$$

$$-A_{n2}k\,\mathrm{sh}(kh_2)+B_{n2}k\,\mathrm{ch}(kh_2)+C_{n2}[kh_2\mathrm{sh}(kh_2)-\mathrm{ch}(kh_2)]+D_{n2}[-kh_2\,\mathrm{ch}(kh_2)+\mathrm{sh}(kh_2)]=0, \qquad (4)$$

$$E_2k^2[A_{n2}k\,\mathrm{ch}(kh_2)-B_{n2}k\,\mathrm{sh}(kh_2)-C_{n2}kh_2\mathrm{ch}(kh_2)+D_{n2}kh_2\,\mathrm{sh}(kh_2)]=\frac{2L_2}{k}(-1)^{n+1}, \qquad (5)$$

$$E_1k^2[A_{n1}k\,\mathrm{ch}(kh_1)+B_{n1}k\,\mathrm{sh}(kh_1)C_{n1}kh_1\,\mathrm{ch}(kh_1)+D_{n1}kh_1\,\mathrm{sh}(kh_1)]=\frac{2L_1}{K}(-1)^{n+1}, \qquad (6)$$

$$\frac{E_1}{1-\nu_1^2}\left[\frac{\nu_1}{4(\nu_1-1)}(K_1+L_1h_1)-\frac{R_1(T_0-T_1)}{2\nu_1h_1}-\frac{\nu_1(\nu_1-2)}{4(\nu_1-1)\mu_1}(K_1+L_1h_1)\right]=\alpha_T^1T_1, \qquad (7)$$

$$A_{n1}k\,\mathrm{sh}(kh_1)+B_{n1}k\,\mathrm{ch}(kh_1)+C_{n1}[kh_1\,\mathrm{sh}(kh_1)-\mathrm{ch}(kh_1)]+D_{n1}[kh_1\,\mathrm{ch}(kh_1)-\mathrm{sh}(kh_1)]=0, \qquad (8)$$

$$k[B_{n1}k+C_{n1}-B_{n2}k-C_{n2}]=\left[\frac{\nu_2-2}{\nu_2-1}\frac{K_2}{\mu_2}-\frac{\nu_2-2}{\nu_2-1}\frac{K_1}{\mu_1}\right]\frac{1}{2k}(-1)^{n+1}, \qquad (9)$$

$$k[A_{n1}k-2D_{n1}+2D_{n2}-A_{n2}k]=\left[\frac{3\nu_1-2}{8(\nu_1-1)\mu_i}L_1-\frac{3\nu_2-2}{8(\nu_2-1)\mu_2}L_2\right]\frac{(-1)^{n+1}}{k^2}, \qquad (10)$$

$$\frac{L_1}{\mu_1}\,\frac{3\nu_1-2}{\nu_1-1}=\frac{3\nu_2-2}{\nu_2-1}\,\frac{L_2}{\mu_2}, \qquad (11)$$

$$B_{n1}k-C_{n1}=B_{n2}k-C_{n2}, \qquad (12)$$

$$\frac{E_1}{1-\nu_1^2}\left[\frac{\nu_1}{4(\nu_1-1)}K_1-\frac{R_1(T_0-T_1)}{2\nu_1h_1}-\frac{\nu_1\nu_1-2}{4(\nu_1-1)\mu_i}K_1\right] \qquad (13)$$

$$=\frac{E_2}{1-\nu_2^2}\left[\frac{\nu_2}{4(\nu_2-1)}K_2+\frac{R_2(T_0-T_2)}{2\nu_2h_2}-\frac{\nu_2\nu_2-2}{4(\nu_2-1)\mu_2}K_2\right],$$

$$k^3[E_1A_{n1}-E_2A_{n2}]=\frac{2}{k}(-1)^{n+1}[L_1-L_2]. \quad (14)$$

Equations (3)-(14) are a system of linear algebraic equations for determining the unknown coefficients A_{n1}, ..., L_{12}. After they have been found, the true stresses in the structure are calculated by the formulas:

$$\sigma_{yy}^1=\sigma_{yy}^{1*}-\left(\frac{T_1-T_0}{h_1}y+T_0\right)\alpha_T^1,\ \sigma_{yy}^2=\sigma_{yy}^{2*}-\left(\frac{T_0-T_2}{h_2}y+T_0\right)\alpha_T^2;$$

$$\sigma_{xx}^1=\sigma_{xx}^{1*}-\left(\frac{T_1-T_0}{h_1}y+T_0\right)\alpha_T^1,\ \sigma_{xx}^2=\sigma_{xx}^{2*}-\left(\frac{T_0-T_2}{h_2}+T_0\right)\alpha_T^2;$$

$$\sigma_{xy}^1=\sigma_{xy}^{1*},\ \sigma_{xy}^2=\sigma_{xy}^{2*}.$$

The numerical calculation by the suggested procedure was carried out for a structure whose boxes were made of steel 20, and the cladding sheet of steel St3. The results of the numerical calculation are presented in Figs. 2-4. The presented calculation makes it possible to check the heat treatment regimes of composite structures, and it is of practical interest when the equipment for the forming of flat concrete slabs is designed, as well as in other branches of engineering.

NOTATION

q_1, q_2, q_3, intensities taking into account the effect of the influence of the temperature (q_1 and q_2), of the stiffening ribs (q_3); b, distance between the stiffening ribs; h_1, thickness of the cladding layers; h_2) thickness of the inner layers; 2s, thickness of the stiffening ribs; λ_T, thermal conductivity; α, coefficient of linear expansion; x, y, Cartesian coordinates, mm; λ, μ, Lame constants; ν, Poisson ratio; E, modulus of elasticity; G, shear modulus; T, temperature; t, time; Δ, Laplace operator; u, v, vectors of displacement of points of the body; e_{xx}, e_{yy}, strain components; σ_{xx}, σ_{yy}, σ_{xy}, stress components; n, number of terms of a series; A_{ni}, B_{ni}, C_{ni}, D_{ni}, K_i, L_i, unknown integration constants; i = 1, 2, subscripts relating to the cladding and inner layers.

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